

# The Bio-heat Equation

How to model heating of  
tissue during laser treatment  
irradiation

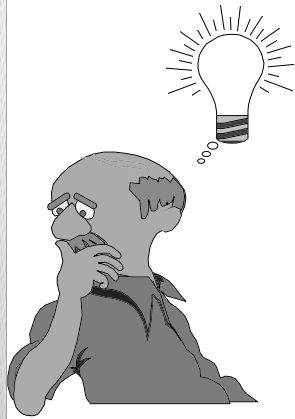
Medical Optics

## Why Thermal Models?

- During all medical laser applications based on heating, it is desirable to have complete knowledge of the temperature distribution.
- Invasive temperature measurements can provide information of the tissue temperature at discrete points only.
- The number of probes that can be implanted is restricted by patient tolerance and practical aspects and thus the tissue temperature knowledge is limited.

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# Motivation for a mathematical model



A mathematical model for calculating the tissue temperature during the treatment could in a valuable way complement the invasive temperature measurements.

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## Some motivations for a model

- A treatment can be optimised with respect to the heat application geometry by maximising the therapeutic effect while minimising unwanted side effects
- The outcome of a treatment can be evaluated based on the model predictions
- It can also be used for extensive parametric studies in order to characterise the stability of various treatment parameters
- New treatment strategies can be suggested and evaluated

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# Heating of tissue using laser light



To better understand the heating of tissue during laser irradiation we need to look more in detail on the thermal properties of tissue

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## Heat flow factors

Among the general factors to consider to understand transfer of heat in tissue are

- The thermophysical properties of the tissue (heat capacity, thermal conductivity etc)
- Geometry of the irradiated organism
- Heat production due to absorption of laser light
- Heat production due to metabolic processes
- Heat flow due to perfusion of blood
- Thermoregulatory mechanisms

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# Conservation of Energy

The balance of thermal energy in a small “control volume” can be stated:

$$q_{gain} = q_{storage} + q_{loss} + W$$

where the terms are the rates of heat gained by light absorption and from the surrounding control volumes, stored by the tissue, lost through the boundary of the volume and W work performed by the tissue and metabolic heating

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## Consider the boundary separately

To establish an understanding for the problem we need to divide the problem in two parts:

- flow of heat between control volumes inside the tissue without any boundary effects
- consider the boundary conditions for control volumes at the tissue surface

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# Mechanisms of heat flow

The two main mechanisms for heat flow inside a tissue is through conduction, meaning that the gradient in temperature within the tissue itself drives the flow, and through convection of thermal energy by the perfusing blood.

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# Interior thermal flow

The conducted heat flow is governed by the Fourier law of heat conduction.

The law states that the amount of thermal energy conducted through a medium is proportional to the cross sectional area, the temperature difference and the length of time.

It is inversely proportional to the length across the medium.

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# The Fourier Law of Heat Conduction

$$Q = -kA(T_2 - T_1)\Delta t / \Delta L$$

Heat flow is the rate of heat conducted per unit area per unit time, according to

$$f = Q / (A \Delta t) = -k(T_2 - T_1) / \Delta L$$

This means in the limiting case for infinitesimally small  $\Delta L$ :  $\mathbf{f} = -k\nabla T$

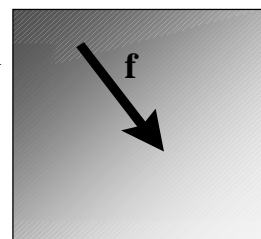
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## Flow of Heat in Tissue

The flow of heat due to thermal conduction can be expressed by

$$\mathbf{f} = -k\nabla T$$

- where  $\mathbf{f}$  is the heat flux vector,
- $k$  is the coefficient of heat conductivity, and
- the vector quantity  $-\nabla T$  has a magnitude equal to maximum change in temperature per unit distance



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# Interior Heat Flow

The two major paths for thermal flow inside the tissue is thermal conduction and through the blood flow. The net loss for a control volume is thus:

$$q_{loss} - q_{gain,cond} = -q_{storage} + S$$

$$\Leftrightarrow$$

$$\nabla \cdot \mathbf{f} = -\rho c \frac{\partial T}{\partial t} + q_p + S$$

where  $\rho$  is the density ( $\text{kg m}^{-3}$ ),  $c$  is the specific heat ( $\text{J kg}^{-1} \text{K}^{-1}$ ) and  $q_p$  is the rate of heat flow by blood

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# The Bio-Heat Equation

This can be written as the Bio-heat Equation



$$\rho c \frac{\partial T}{\partial t} = \nabla(k \nabla T) + q_s + q_p + q_m$$

with sources due to absorbed laser light, blood perfusion and metabolic activity, respectively.

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# Thermal properties of tissue

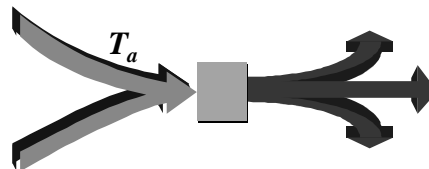
Thermo-physical properties of human tissue and water (adapted from *AF Emery and KM Sekins* (1982); *K Giering et al.* (1995))

Material	Conductivity (W m <sup>-1</sup> K <sup>-1</sup> )	Density (kg m <sup>-3</sup> )×10 <sup>-3</sup>	Specific heat (kJ kg <sup>-1</sup> K <sup>-1</sup> )	Diffusivity (m <sup>2</sup> s <sup>-1</sup> ×10 <sup>7</sup> )
Muscle	0.38-0.54	1.01-1.05	3.6-3.8	0.90-1.5
Fat	0.19-0.20	0.85-0.94	2.2-2.4	0.96
Kidney	0.54	1.05	3.9	1.3
Heart	0.59	1.06	3.7	1.4
Liver	0.57	1.05	3.6	1.5
Brain	0.16-0.57	1.04-1.05	3.6-3.7	0.44-1.4
Water @ 37 °C	0.63	0.99	4.2	1.5

# Blood perfusion

The blood transports thermal energy both in and out from the control volume studied. The net inflow can be written as:

$$q_p = -\omega_b \rho_b c_b \rho (T - T_a)$$



where  $\omega$  is the blood perfusion (volume blood per unit mass tissue per unit time, 1/s),  $\rho_b$  and  $c_b$  are the density and specific heat of the blood, and  $T_a$  is the temperature of the arterial blood

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## Integration over a control volume

The bioheat equation can be solved numerically using the control volume formulation. By requiring that the equation holds for a finite volume and by assuming that the metabolic heat generation can be neglected, we obtain

$$V\rho c \frac{\partial \bar{T}}{\partial t} = - \iint_s k \nabla T \cdot \vec{n} \, ds - V\omega \rho_b c_b \rho (\bar{T} - T_a) + V\bar{q}$$

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## Boundary Conditions

At the boundaries other thermal energy losses also exists.

- **Radiation** is energy transfer via electromagnetic wave action
- **Convection** is transfer of thermal energy through a fluid due to bulk motion of the fluid
- **Evaporation** requires energy for the phase transition



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# Radiation



No medium is necessary for energy transfer, since it is transported by electromagnetic radiation.



Thermal radiation is emitted mainly in the visible and NIR wavelength region at the expenses of the internal energy of the system.

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## The Stefan-Boltzmann Law

The total emitted thermal energy of a body over the entire emission spectrum is given by

$$E(T) = \varepsilon \sigma T^4 \quad [\text{W/m}^2]$$

where  $\sigma$  is the Stefan-Boltzmann constant

$$\sigma = 5.67 \cdot 10^{-8} \quad [\text{W}/(\text{m}^2 \text{ K}^4)],$$

and  $\varepsilon$  is the thermal emissivity averaged over the thermal radiation spectrum.

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# Radiation from tissue

In most modelling of internal biological tissues, the contribution from intrinsic radiative heat transfer processes is negligible, and it will most often not be considered in the heat balance equation.



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# Convection



Convection is difficult to model both for laminar and turbulent flow. Often empirically based relations are needed to model the energy transport. This is due to several simultaneous processes of heat flow between the media.

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## Heat flow due to Convection

Convection is modelled according to Newton's law of cooling:

$$q_c = h_c(T_e - T_s)$$

where  $q_c$  is the heat flux at the surface into the tissue due to convection ( $\text{W m}^2$ ),  $h_c$  is the convection heat transfer coefficient ( $\text{W m}^2 \text{ K}^{-1}$ ),  $T_e$  and  $T_s$  are the environmental and surface temperatures ( $^{\circ}\text{C}$ ), respectively.

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## Convection of air

The convection heat transfer coefficient can for a quadratic flat surface be expressed as:

$$h_c = Nu k_f / D$$

where  $Nu$  is the Nusselt number (dimensionless),  $k_f$  is the thermal conductivity of air ( $\text{W m}^{-1} \text{ K}^{-1}$ ) and  $D$  is a characteristic length equal to the length of one side of the square plate (m).

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# The Nusselt Number

The Nusselt number under free convection is empirically found to be

$$Nu = 0.54[g_{loc} \beta(T_s - T_e)D^3/\nu^2]^{0.25}$$

where  $g_{loc}$  is the local acceleration due to gravity ( $\text{m s}^{-2}$ ),  $\beta$  is the volume coefficient of expansion of air ( $\text{K}^{-1}$ ) evaluated at the mean value of the surface and environmental temperatures and  $\nu$  is the kinematic viscosity of air ( $\text{m}^2 \text{s}^{-1}$ ).

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# Water evaporation



Heat loss due to evaporation of water at a laser-irradiated surface can be modelled as :

$$q_{\text{evap}} = h_{\text{fg}} h_{\text{m}} (\rho_{\text{v,e}} - \rho_{\text{v,sat}})$$

where  $q_{\text{evap}}$  is the heat flux into the tissue due to water evaporation ( $\text{W m}^{-2}$ ),  $\rho_{\text{v,e}}$  is the density of water vapour in air at room temperature ( $\text{kg m}^{-3}$ ),  $\rho_{\text{v,sat}}$  is the mass density of saturated water vapour ( $\text{kg m}^{-3}$ ) at the surface temperature  $T_s$ ,  $h_{\text{fg}}$  is the phase-change enthalpy of water ( $\text{J kg}^{-1}$ ) and  $h_{\text{m}}$  is the convection mass transfer coefficient ( $\text{m s}^{-1}$ ).

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# Convection mass transfer

The convection mass transfer coefficient can be expressed by

$$h_m = h_c / (\rho_a c_a Le^{2/3})$$

where  $h_c$  is the convection heat transfer coefficient,  $\rho_a$  and  $c_a$  are the density ( $\text{kg m}^{-3}$ ) and specific heat of air ( $\text{J kg}^{-1} \text{K}^{-1}$ ), respectively, and  $Le$  is the Lewis number (dimensionless) for the diffusion of water vapour into air.

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# Heat flux through surface

The total heat flux  $q_{\text{surf}}(i,0)$  ( $\text{Wm}^{-2}$ ) at the upper bounding surface into the surface control volume  $(i,0)$  can now be modelled as



$$q_{\text{surf}}(i,0) = h_c(T_e - T_{i,0}) + \sigma \epsilon (T_e^4 - T_{i,0}^4) + h_c h_{fg} (\rho_{v,e} - \rho_{v,\text{sat}}) / \rho_a c_a Le^{2/3}$$

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